Subdivision is an algorithmic technique to generate smooth surfaces as a sequence of successively refined polyhedral meshes. Its origins go back to 1978 when Catmull and Clark, and Doo and Sabin first proposed to generalize spline-patch methods to meshes of arbitrary topology. Subdivision algorithms are exceptionally simple, work for arbitrary control meshes and produce globally smooth surfaces. Special choices of subdivision rules allow for the introduction of features into a surface in a simple way. Subdivision-based representations of complex geometry can be manipulated and rendered very efficiently, which makes subdivision a highly suitable tool for interactive animation and modeling systems.

**Basic approach at a glance**

Start: Control mesh  
Process: Iterated application of refinement rules  
Result: Smooth curve or surface (after an infinite number of iterations)

**Subdivision Curve** (4-point-scheme):

Start with 4 points in plane connected with straight line segments  
In each step 3 new points are inserted in-between the old points.  
The new points are computed by taking a weighted average of nearby old points: two to the left and two to the right with weights $1/16*(-1,9,9-1)$ respectively (ignoring the boundaries)

Connecting them results in piecewise linear curve. After an infinite number of iterations, it converges against a parametric curve. To prove convergence and continuity, subdivision is expressed as an iterated matrix-vector-multiplication:

$$\left(\begin{array}{c}
p_{3}^{j+1} \\
p_{2}^{j+1} \\
p_{1}^{j+1} \\
p_{0}^{j+1}
\end{array}\right) = \frac{1}{16} \left(\begin{array}{cccc}
-1 & 9 & 9 & -1 \\
0 & 0 & 16 & 0 \\
0 & -1 & 9 & 9 \\
0 & 0 & 16 & 0
\end{array}\right) \left(\begin{array}{c}
p_{3}^{j} \\
p_{2}^{j} \\
p_{1}^{j} \\
p_{0}^{j}
\end{array}\right)$$

This representation is not suitable for implementation, due to poor performance!
Subdivision Surfaces:

The generalization to the 3 dimensional case leads to Subdivision Surfaces.

Each triangle is split into 4 new triangles by Insertion of new points in the center of every edge.

Local subdivision:

Exclude some edges from refinement => sharp creases. Use sharpness parameter to define, how often an edge is excluded from refinement => arbitrary sharpness/smoothness
Classification of Subdivision Schemes:
- Mesh type: Triangles, Quads
- Smoothness of the limit surface
- Refinement rule
  a) Face split:
  Each face of the original mesh is split into four, by inserting new vertices on the edges.

  For quads, an additional point is created for each face.

  Depending on whether old vertices may be moved or not, a scheme is called approximating, otherwise interpolating.

  b) Vertex split:
  Old vertices are replaced by some new vertices (one for each face adjacent to the old one)
Common Algorithms:
Approximating face split:
  - Loop (Triangular meshes)
  - Catmull-Clark (Quad meshes)
Interpolating face split:
  - Butterfly (Triangular meshes)
  - Kobbelt (Quad meshes)
Vertex split:
  - Doo-Sabin
  - Midedge

Further reading

A Quick Introduction to Subdivision Surfaces:
http://www.holmes3d.net/graphics/subdivision/

SIGGRAPH 1998 Subdivision course:
http://www.multires.caltech.edu/teaching/courses/subdivision/

Subdivision Surfaces in Character Animation;
http://www.cs.rutgers.edu/~decarlo/readings/derose98.pdf