Exercise 1: Copy-Constant Analysis
Consider the following program where \( x \) is a global and \( y \) is a local variable:

\[
\begin{align*}
\text{main:} & \quad 0 \quad 1 \quad y = 5 \\
& \quad 2 \quad x = 3 \\
& \quad 3 \quad f() \\
& \quad 4 \quad x = 7 \\
& \quad 5 \quad f() \\
\text{f():} & \quad 6 \quad y = 7 \\
& \quad 7 \quad y = x \\
& \quad 8 \quad x = y \\
& \quad 9 \\
\text{main():} & \quad 0 \quad y = 5 \\
& \quad 1 \quad x = 3 \\
& \quad 2 \quad f() \\
& \quad 3 \quad x = 7 \\
& \quad 4 \quad f() \\
& \quad 5
\end{align*}
\]

a) Using Copy-Constants analysis (slides 558-560), compute the summary (transformation) \( M \in M \) for the procedure \( f() \).

b) Now compute the transfer function for the calls to the procedure \( f() \) using your summary, i.e. compute \( H(M) \).

c) Having computed the transfer function for calls to the procedure \( f() \), write out a constraint system \( \mathcal{R} \) for Constant Propagation Analysis of the above program and solve it.

Solution:

a) For the pp.6 we used \( y \mapsto 0 \) in the tutorial, i.e. applied the definition of enter\(^i\). In order to abstract from this definition, let us use identity function for all variables to compute the transformation \( M \). This won’t affect the resulting transformation, but makes nicer separation of concerns.

<table>
<thead>
<tr>
<th></th>
<th>( { x \mapsto x, y \mapsto y } )</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>( { x \mapsto x, y \mapsto y } )</td>
</tr>
<tr>
<td>7</td>
<td>( { x \mapsto x, y \mapsto 7 } )</td>
</tr>
<tr>
<td>8</td>
<td>( { x \mapsto x, y \mapsto x } \cup { x \mapsto x, y \mapsto 7 } = { x \mapsto x, y \mapsto 7 \sqcup x } )</td>
</tr>
<tr>
<td>9</td>
<td>( { x \mapsto 7 \sqcup x, y \mapsto 7 \sqcup x } )</td>
</tr>
</tbody>
</table>
b)  

\[ H(M) = \text{Id}_{\text{Locals}} \oplus (M \circ \text{enter}^5)|_{\text{Globals}} \]

\[ = \{ y \mapsto y \} \oplus (\{ x \mapsto 7 \sqcup x, y \mapsto 7 \sqcup x \} \circ \{ x \mapsto x, y \mapsto 0 \})|_{\text{Globals}} \]

\[ = \{ y \mapsto y \} \oplus (\{ x \mapsto 7 \sqcup x, y \mapsto 7 \sqcup x \})|_{\text{Globals}} \]

\[ = \{ y \mapsto y \} \oplus \{ x \mapsto 7 \sqcup x \} \]

\[ = \{ x \mapsto 7 \sqcup x, y \mapsto y \} \]

c)  

\[ \mathcal{R}[0] \supseteq \text{enter}^4 d_0 \]

\[ \mathcal{R}[1] \supseteq [x = 3]^3(\mathcal{R}[0]) \]

\[ \mathcal{R}[2] \supseteq [x = 3]^3(\mathcal{R}[1]) \]

\[ \mathcal{R}[3] \supseteq H([f]^3)(\mathcal{R}[2]) \]

\[ \mathcal{R}[4] \supseteq [x = 7]^3(\mathcal{R}[3]) \]

\[ \mathcal{R}[5] \supseteq H([f]^3)(\mathcal{R}[4]) \]

\[ \mathcal{R}[6] \supseteq \mathcal{R}[f] \]

\[ \mathcal{R}[7] \supseteq [y = 7]^3(\mathcal{R}[6]) \]

\[ \mathcal{R}[8] \supseteq [y = x]^3(\mathcal{R}[7]) \]

\[ \mathcal{R}[9] \supseteq [x = y]^3 \]

\[ \mathcal{R}[0] = \{ x \mapsto \top, y \mapsto 0 \} \]

\[ \mathcal{R}[1] = \{ x \mapsto \top, y \mapsto 5 \} \]

\[ \mathcal{R}[2] = \{ x \mapsto 3, y \mapsto 5 \} \]

\[ \mathcal{R}[3] = \{ x \mapsto \top, y \mapsto 5 \} \]

\[ \mathcal{R}[4] = \{ x \mapsto 7, y \mapsto 5 \} \]

\[ \mathcal{R}[5] = \{ x \mapsto 7, y \mapsto 5 \} \]

Exercise 2: Sharir/Pnueli, Cousot

Consider the following program (all variables are globals):
The main procedure is analyzed in the context $d_0 = \{ x \mapsto \top, y \mapsto \top \}$. The procedure $y2x$ is analyzed in the context $[3, d_0]^2$ and $[5, d_0]^2$.

a) Calculate the value for $[3, d_0]^2$.
b) Analyze $y2x$ by calculating $[12, [3, d_0]^2]^i$.
c) Calculate the value for $[5, d_0]^2$.
d) Analyze $y2x$ by calculating $[12, [5, d_0]^2]^i$.
e) Calculate the value for $[6, d_0]^2$.

Solution:

$$[1, d_0]^2 = \{ x \mapsto \top, y \mapsto \top \}$$
$$[2, d_0]^2 = \{ x \mapsto 0, y \mapsto \top \}$$
$$[3, d_0]^2 = \{ x \mapsto 0, y \mapsto 5 \}$$
$$[11, [3, d_0]^2]^2 = [3, d_0]^2$$
$$[12, [3, d_0]^2]^2 = \{ x \mapsto 5, y \mapsto 5 \}$$
$$[4, d_0]^2 = \{ x \mapsto 5, y \mapsto 5 \}$$
$$[5, d_0]^2 = \{ x \mapsto 5, y \mapsto 10 \}$$
$$[11, [5, d_0]^2]^2 = [5, d_0]^2$$
$$[12, [5, d_0]^2]^2 = \{ x \mapsto 10, y \mapsto 10 \}$$
$$[6, d_0]^2 = \{ x \mapsto 10, y \mapsto 10 \}$$

Exercise 3: Call-String Approach

Consider the following program where $x$ is a global and $y$ is a local variable:

Using the Call-String approach, analyse the program with
a) $d = 0$ (here, it means empty call stack) and  

b) $d = 1$ (not more than one call)  

and give the result at the end of main() for each case. 

c) Does a call stack $d \geq 2$ improve the solution? 

Solution:  

a) We abbreviate by $R[u]$ the unknown $R[u, \langle \rangle]$ where $\langle \rangle$ denotes the empty sequence. 

We set up the following constraint system: 

$R[0] \supseteq \{ x \mapsto \top, y \mapsto 0 \}$  
$R[1] \supseteq [y = 5] (R[0]) = R[0] \oplus \{ y \mapsto 5 \}$  
$R[2] \supseteq [x = 3] (R[1]) = R[1] \oplus \{ x \mapsto 3 \}$  
$R[3] \supseteq \text{combine}^*(R[2], R[9])$  
$R[4] \supseteq [x = 7] (R[3]) = R[3] \oplus \{ x \mapsto 7 \}$  
$R[5] \supseteq \text{combine}^*(R[4], R[9])$  
$R[6] \supseteq \text{enter}^2(R[2]) \sqcup \text{enter}^4(R[4])$  
$R[7] \supseteq [y = 7] (R[6]) = R[6] \oplus \{ y \mapsto 7 \}$  
$R[8] \supseteq [y = x] ^2 (R[7]) \sqcup [y] ^2 (R[7]) = \left(R[7] \oplus \{ y \mapsto R[7](x) \}\right) \sqcup R[7]$  
$R[9] \supseteq [x = y] ^2 (R[8]) = R[8] \oplus \{ x \mapsto R[8](y) \}$  

To solve the system, we would need to perform the round-robin algorithm, as there are cyclic dependencies between the constraints. That is, a single iteration is not sufficient (need 3 in this case). Here we write down the solution directly. 

<table>
<thead>
<tr>
<th>$u$</th>
<th>$R[u]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>${ x \mapsto \top, y \mapsto 0 }$</td>
</tr>
<tr>
<td>1</td>
<td>${ x \mapsto \top, y \mapsto 5 }$</td>
</tr>
<tr>
<td>2</td>
<td>${ x \mapsto 3, y \mapsto 5 }$</td>
</tr>
<tr>
<td>3</td>
<td>${ x \mapsto \top, y \mapsto 5 }$</td>
</tr>
<tr>
<td>4</td>
<td>${ x \mapsto 7, y \mapsto 5 }$</td>
</tr>
<tr>
<td>5</td>
<td>${ x \mapsto \top, y \mapsto 5 }$</td>
</tr>
<tr>
<td>6</td>
<td>${ x \mapsto \top, y \mapsto 0 }$</td>
</tr>
<tr>
<td>7</td>
<td>${ x \mapsto \top, y \mapsto 7 }$</td>
</tr>
<tr>
<td>8</td>
<td>${ x \mapsto \top, y \mapsto \top }$</td>
</tr>
<tr>
<td>9</td>
<td>${ x \mapsto \top, y \mapsto \top }$</td>
</tr>
</tbody>
</table>

Therefore, at program exit (node 5) we have the following information: $\{ x \mapsto \top, y \mapsto 5 \}$. 

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b) We abbreviate by $R[u,v]$ the unknown $R[u,\langle v\rangle]$ where $\langle v\rangle$ denotes the sequence of length 1 consisting of the element $v$, where $(v,f();w)$ is the edge with procedure call.

We set up the following constraint system:

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R[0] \vdash { x \mapsto \top, y \mapsto 0 }$</td>
<td>${ x \mapsto \top, y \mapsto 0 }$</td>
</tr>
<tr>
<td>$R[1] \vdash { y = 5 }^\sharp(R[0])$</td>
<td>${ x \mapsto \top, y \mapsto 5 }$</td>
</tr>
<tr>
<td>$R[2] \vdash { x = 3 }^\sharp(R[1])$</td>
<td>${ x \mapsto 3, y \mapsto 5 }$</td>
</tr>
<tr>
<td>$R[3] \vdash \text{combine}^\sharp(R[2], R[9,2])$</td>
<td>${ x \mapsto \top, y \mapsto 5 }$</td>
</tr>
<tr>
<td>$R[4] \vdash { x = 7 }^\sharp(R[3])$</td>
<td>${ x \mapsto \top, y \mapsto 5 }$</td>
</tr>
<tr>
<td>$R[5] \vdash \text{combine}^\sharp(R[4], R[9,4])$</td>
<td>${ x \mapsto 7, y \mapsto 5 }$</td>
</tr>
<tr>
<td>$R[6,2] \vdash \text{enter}^\sharp(R[2])$</td>
<td>${ x \mapsto 3, y \mapsto 0 }$</td>
</tr>
<tr>
<td>$R[7,2] \vdash { y = 7 }^\sharp(R[6,2])$</td>
<td>${ x \mapsto 3, y \mapsto 7 }$</td>
</tr>
<tr>
<td>$R[8,2] \vdash { y = x }^\sharp(R[7,2]) \sqcup { ; }^\sharp(R[7,2])$</td>
<td>${ x \mapsto 3, y \mapsto \top }$</td>
</tr>
<tr>
<td>$R[9,2] \vdash { x = y }^\sharp(R[8,2])$</td>
<td>${ x \mapsto \top, y \mapsto \top }$</td>
</tr>
<tr>
<td>$R[6,4] \vdash \text{enter}^\sharp(R[4])$</td>
<td>${ x \mapsto 7, y \mapsto 0 }$</td>
</tr>
<tr>
<td>$R[7,4] \vdash { y = 7 }^\sharp(R[6,4])$</td>
<td>${ x \mapsto 7, y \mapsto 7 }$</td>
</tr>
<tr>
<td>$R[8,4] \vdash { y = x }^\sharp(R[7,4]) \sqcup { ; }^\sharp(R[7,4])$</td>
<td>${ x \mapsto 7, y \mapsto 7 }$</td>
</tr>
<tr>
<td>$R[9,4] \vdash { x = y }^\sharp(R[8,4])$</td>
<td>${ x \mapsto 7, y \mapsto 7 }$</td>
</tr>
</tbody>
</table>

Therefore, at program exit (node 5) we have the following precise information: $\{ x \mapsto 7, y \mapsto 5 \}$.

Therefore, $d \geq 2$ does not provide more information than $d = 1$. 

c) We do not have recursive function calls nor do we have nested function calls of depth two or more which is depicted by the call-graph.

![Call-graph](call-graph.png)