Exercise 1: VLIWs slot filling

Consider the following basic block:

```
x = 10;
y = 15;
z = M[x+y];
x = 2*z;
y = 20;
M[z] = x+y;
```

1. Draw the Data Dependency Graph of this program.
2. Rewrite the program so that there are no DD and UD edges.
3. Write down the rewritten program as a sequence of VLIWs with 2 slots.

Solution:

1. Let \( S_1, \ldots, S_6 \) be the 6 program statements.

2. We put the program in SSA form:

```
x = 10;
y = 15;
z = M[x+y];
x1 = 2*z;
y1 = 20;
M[z] = x1+y1;
```
The Data Dependency Graph becomes:

3. Empty slots represent **nop** instructions.

<table>
<thead>
<tr>
<th>Word</th>
<th>State</th>
</tr>
</thead>
<tbody>
<tr>
<td>x=10; y=15;</td>
<td>0 0 0 0 0</td>
</tr>
<tr>
<td>z = M[x+y]; y1 = 20;</td>
<td>0 0 1 0 0</td>
</tr>
<tr>
<td>x1 = 2*z; M[z] = x1+y1;</td>
<td>0 0 0 0 0</td>
</tr>
</tbody>
</table>

**Exercise 2: Solving Difference Constraints**

Solve the following ILP problem:

\[
\begin{align*}
    x & \leq 15 \\
    y & \leq 20 \\
    x + 2 & \leq y \\
    z + 3 & \leq x \\
    3 & \leq y \\
    z + 5 & \leq y \\
    10 & \leq z \\
    y - 5 & \leq z 
\end{align*}
\]

**Solution:** We first check that it is possible to draw a graph for the system of inequalities:

- there are at most 2 variables per inequality
- variables have no scaling factors

Both conditions are satisfied, thus, we can build a graph.
The graph has no strictly positive cycles, so we can apply Bellman-Ford algorithm.

\[
\begin{array}{c|ccc}
\text{x} & -\infty & -\infty & 13 & 13 \\
\text{y} & -\infty & 3 & 15 & 15 \\
\text{z} & -\infty & 10 & 10 & 10 \\
\end{array}
\]

Finally, we check that the upper-bound constraints are satisfied: \( x = 13 \leq 15 \) and \( y = 15 \leq 20 \).

Thus \( x = 13, y = 15, y = 10 \) is a solution to the inequality system.