Containment Problems for Polynomial Zonotopes

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Background

For two sets U and V, checking whether $U \subseteq V$ defines a class of problems, called containment problems. Although in general these problems cannot be solved algorithmically, containment problems are often solvable if some structure for U and V is assumed, e.g., when U and V are polytopes, zonotopes, ellipsoids, ...

Containment problems are used for reachability analysis, set-based observers, fault detection, robust control, controller synthesis, and conformance checking. Currently, containment checks are performed mostly on zonotopes or polytopes, since these are typically the only sets that are involved for computations. However, in the field of reachability analysis, new methods have been developed that make use of new set-representations which can describe non-convex sets, such as polynomial zonotopes and constrained polynomial zonotopes [1]. Due to this non-convexity, however, many properties that were easy to check for zonotopes or polytopes become more intricate and difficult, and a standard solution to avoid this problem is to over-approximate these sets by zonotopes, and then check the property in question.

This is also the case for containment, i.e., one way to check for containment is to split a polynomial zonotope, over-approximate each part with a zonotope, and then check for containment. Due to this over-approximation, significant errors can be made, and it is therefore of interest whether a better method can be found.

Description

The focus of this thesis is the containment problem for these new classes of sets, i.e., polynomial zonotopes, constrained polynomial zonotopes,... For zonotopes, containment was proven to be co-NP-complete [2], while for polytopes this varies depending on the type of representation. Therefore, an analysis of the complexity of the problem for polynomial zonotopes is crucial, in order to estimate whether a fast and exact algorithm can exist.

Concretely, the goal of this thesis is twofold: First, the complexity of the problem will be determined; if you have some basic knowledge of complexity theory, but are not sure how to show that a specific problem is (co-)NP-hard for example, this is the perfect opportunity to learn this rather abstract side of numerical analysis. Then, an algorithm to approximately solve the containment problem will be developed. All programming will be done in Matlab, and the final implementation of the approaches should be integrated into the CORA toolbox so that it can be made publicly available in the next CORA release.

Tasks

- Literature review on the topic of containment problems
- Analysis of the complexity of the containment problem for polynomial zonotopes
- Implementation of an approximate algorithm
- Evaluation of the performance by comparing the result to the currently implemented method in CORA
- Integration of the final implementation into the CORA toolbox
- *Optional:* Extension of the approach to constrained polynomial zonotopes, and possible more diverse types of set-representations, such as ellipsoids,...

References

[1] Niklas Kochdumper and Matthias Althoff. Constrained polynomial zonotopes, 2020.

[2] A. Kulmburg and M. Althoff. On the co-np-completeness of the zonotope containment problem. *European Journal of Control*, To appear.

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Research project:
justITSELF

Type:
Bachelor Thesis

Research area:
Containment Problems,
Reachability Analysis

Programming language:
MATLAB

Required skills:
Good mathematical background. A basic understanding of complexity theory may be useful, but can be learned along the way.

Language:
English

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