

Approximations to the Zonotope Containment Problem



Technische Universität München

Background

For two sets U and V , deciding whether $U \subseteq V$ defines a class of decision problems called containment problems. Although in general these problems cannot be solved algorithmically, containment problems are often solvable if some structure for U and V is assumed, e.g., when U and V are polytopes or zonotopes. Zonotopes are sets that cannot only be represented compactly but are also closed under linear maps and Minkowski sum. Owing to these favourable properties, zonotopes are used for reachability analysis, set-based observers, fault detection, robust control, controller synthesis, and conformance checking. The aforementioned applications often require solving the zonotope containment problem, i.e., whether a zonotope is contained in another one. This is, for instance, useful for verifying an invariant of a discrete-time system by checking whether the reachable set of the next step is contained in the previous one.

Recently, the containment problem for zonotopes was shown to be co-NP-complete [1], meaning that, unless $P=NP$, no polynomial-time algorithm solving this problem can exist. Algorithms that solve the problem approximately exist [2], but have a fixed accuracy that can not be tuned.

Description

The goal of this thesis is to find new ways to solve the zonotope containment problem approximately. This can be achieved either by improving the already implemented methods in CORA, or by finding completely new algorithms that solve the problem stochastically for example.

All programming will be done in Matlab, and the final implementation of the approaches should be integrated into the CORA toolbox so that it can be made publicly available in the next CORA release.

Tasks

- Literature review on the topic of containment problems
- Development and implementation of one or several new algorithms for the zonotope containment problem, in particular a stochastic algorithm
- Evaluation of the performance by comparing the result to the currently implemented method in CORA
- Integration of the final implementation into the CORA toolbox
- *Optional*: Extension of the approach to similar set-representations, such as constrained zonotopes, ellipsoids,...

References

- [1] A. Kulmburg and M. Althoff. On the co-np-completeness of the zonotope containment problem. *European Journal of Control*, To appear.
- [2] S. Sadraddini and R. Tedrake. Linear encodings for polytope containment problems. In *IEEE 58th Conference on Decision and Control*, pages 4367–4372, 2019.



Fakultät für Informatik

Lehrstuhl für Echtzeitsysteme und Robotik

Supervisor:

Prof. Dr.-Ing. Matthias Althoff

Advisor:

Adrian Kulmburg, M.Sc.

Research project:

justITSELF

Type:

Master Thesis

Research area:

Containment Problems,
Reachability Analysis

Programming language:

MATLAB

Required skills:

Good mathematical background.
A basic understanding of
statistics may be useful.

Language:

English

Date of submission:

22. Juni 2021

For more information please contact us:

Phone:

E-Mail: adrian.kulmburg@tum.de

Internet: www.in.tum.de/i06