Model Checking – Exercise sheet 5

Exercise 5.1
Convert the following Büchi automata with transition-based acceptance condition (“doubled”-transitions have to be seen infinitely often) to equivalent Büchi automata with state-based acceptance conditions. Moreover, give a general procedure to perform this conversion.

(a)

(b)

Exercise 5.2
Extend the set of rules of the LTL to Büchi automata translation to directly deal with the F and G operators.

Exercise 5.3
Let \( \phi = G (\neg p \land F q \lor (q U X q)) \) and \( G \) be a generalized Büchi automaton translated from \( \phi \) using the construction presented in the lecture and the extended set of rules defined in the previous exercise.

(a) Write down the set of subformulae \( Sub(\phi) \).
(b) What is the size of $CS(\phi)$?

(c) How many sets of accepting states does $G$ have?

(d) Is $\{\phi\}$ an accepting state of $G$?

(e) Give a reachable state that has no successors.

(f) Give a successor state of the smallest consistent state containing $\{\phi, q, q U Xq, Fq\}$.

(g) Give a predecessor state of the smallest consistent state containing $\{\phi, q, q U Xq, Fq\}$.

**Exercise 5.4**

Consider the following Büchi automaton $B$:

![Büchi Automaton Diagram]

(a) Give an LTL formula $\phi$ such that $L(B) = [\phi]$.

(b) By using the result from (a), propose a method to construct a Büchi automaton that accepts the complement of the language accepted by $B$.

(c) Construct a Büchi automaton for the formula $G(\neg p \lor (\neg p \mathbf{R} (p \lor \neg q)))$.

(d) By using the result from (c), construct a Büchi automaton that accepts the complement of the language accepted by $B$. 
**Solution 5.1**

The general procedure is as follows.

Let the states of the original automaton be relabeled to \( S \times \{1\} \) and create a copy of the states labeled by \( S \times \{2\} \). For every accepting transition from \((s_1, 1) \rightarrow (s_2, 1)\), change the destination to \((s_2, 2)\). For every non-accepting transition from \((s_1, 2) \rightarrow (s_2, 2)\), change the destination to \((s_2, 1)\).

(a)

(b)

**Solution 5.2**

Extend the definition of NNF to include \( F \) and \( G \), extend the corresponding \( Sub(\phi) \):

- if \( F\phi_1 \in Sub(\phi) \) then \( \phi_1 \in Sub(\phi) \)
- if \( G\phi_1 \in Sub(\phi) \) then \( \phi_1 \in Sub(\phi) \),

and extend rules for transitions as follows: \((M, \sigma, M') \in \Delta \) iff \( \sigma = M \cap AP \) and
• if $F\phi_1 \in \text{Sub}(\phi)$, then $F\phi_1 \in M$ iff $\phi_1 \in M$ or $F\phi_1 \in M'$
• if $G\phi_1 \in \text{Sub}(\phi)$, then $G\phi_1 \in M$ iff $\phi_1 \in M$ and $G\phi_1 \in M'$

Also, the acceptance condition must be extended for $F$: $F$ contains a set $F_\psi$, for every subformula $\psi$ of the form $F\phi_1$, where

$$F_\psi = \{ M \in CS(\phi) | \phi_1 \in M \text{ or } \neg(F\phi_1) \in M \}$$

**Solution 5.3**

Translate $\phi$ into an NNF formula:

$$\phi = G((X(p \lor q) \rightarrow ((\neg p \land Fq) \lor (q \lor Xq)))$$

$$\equiv G((X(\neg p \land \neg q)) \lor ((\neg p \land Fq) \lor (q \lor Xq)))$$

(a) Let $\phi_1 = \neg p \land \neg q$, $\phi_2 = \neg p \land Fq$, and $\phi_3 = q \lor Xq$. We have $\phi = G(X\phi_1 \lor (\phi_2 \lor \phi_3))$ and $\text{Sub}(\phi) = \{ \text{true}, \phi, X\phi_1 \lor (\phi_2 \lor \phi_3), X\phi_1, \phi_2 \lor \phi_3, \phi_1, \phi_2, \phi_3, Fq, Xq, p, q \} \cup \{ \text{false}, \neg \phi, \neg(X\phi_1 \lor (\phi_2 \lor \phi_3)), \neg X\phi_1, \neg(\phi_2 \lor \phi_3), \neg \phi_1, \neg \phi_2, \neg \phi_3, \neg Fq, \neg Xq, \neg p, \neg q \}.$

(b) Only $\phi, X\phi_1, \phi_1, \phi_3, Fq, Xq, p, q$ can independently form consistent states. So, $|CS(\phi)| = 2^8 = 256$ states

(c) $\mathcal{F} = \{F_q \lor Xq, F_F\}$

(d) $\{\emptyset\} \in F_q \lor Xq$ and $\{\emptyset\} \in F_F$

(e) $\{\emptyset\}$ is reachable because it is an initial state, and it has no successors because $X\phi_1 \lor (\phi_2 \lor \phi_3) \not\in \{\emptyset\}$.

(f) $\{\phi, q \lor Xq\}$

(g) $\{\phi, q, q \lor Xq, Fq, Xq\}$

**Solution 5.4**

(a) $\phi = G F(p \land (p \lor (\neg p \land q)))$

(b) Construct a Büchi automaton for $\neg \phi$ by using e.g. the translation in the lecture.

(c) Let $\phi_1 = \neg p \land R(p \lor \neg q)$. Note that $\phi_1, p, q$ are enough to form consistent sets, i.e. we assume that $\phi$ and $\neg p \lor (\neg p \land R(p \lor \neg q))$ are implicitly in every state. So, $CS(\phi) = 2^{\{\phi_1, p, q\}}$. However, we know that $\{p\}$ and $\{p, q\}$ have no successors because of $G$, and $\{\}$ and $\{\phi_1, q\}$ have no successors because of $R$. 

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(d) Notice that $\neg \phi \equiv \mathbf{FG}(\neg p \lor (\neg p \mathbf{R} (p \lor \neg q)))$. It suffices to add a self-looping initial state and transitions from it to all states in (c).