Exercise 6.1
Consider the following Kripke structure $\mathcal{K} = (S, A, \rightarrow, 0, AP, \nu)$, where $A = \{a, b, c, d, e\}$, $AP = \{p\}$, $\nu(6) = \{p\}$, and $\nu(s) = \emptyset$ if $s \neq 6$.

(a) Write down the maximal independence relation $I \subseteq A \times A$.

(b) Write down the maximal invisibility set $U \subseteq A$.

(c) Compute a reduction function $\text{red}$ that satisfies the ample set conditions C0–C3. Whenever possible, choose $\text{red}(s)$ such that it is a proper subset of $en(s)$, for each state $s$.

(d) Use $\text{red}$ to construct a reduced Kripke structure $\mathcal{K}'$ that is stuttering equivalent to the original Kripke structure $\mathcal{K}$. 

Model Checking – Exercise sheet 6
Exercise 6.2

Consider the following Promela model

```promela
byte g;

active proctype m() {
byte x;
m0: x ++;
m1: x ++;
m2: g = x;
}

active proctype n() {
byte y;
n0: y ++;
n1: y ++;
n2: atomic { (g > 0) -> g = g - y }
}

active proctype p() {
p0: atomic { (g > 0) -> g -- }
}
```

and the following properties:

a) The value of g will eventually become one.

b) The process n cannot finish before the process m reaches m1.

For each property, define a labeled Kripke structure with actions extracted from program statements. Determine the independence relation and the invisibility set, and construct a reduced Kripke structure using the ample sets method.
Solution 6.1
(a) \( I = \{ (a, b), (a, c), (a, d), (b, c), (b, e), (c, d), (c, e), (d, e), \\
(b, a), (c, a), (d, a), (c, b), (e, b), (d, c), (e, c), (e, d) \} \)

(b) \( U = \{b, c, d\} \)

(c) \( \text{red}(0) = \{a, b\}, \text{red}(1) = \{c\}, \text{red}(2) = \{a, e\}, \text{red}(5) = \{d\}, \text{red}(4) = \{b, d\}, \text{red}(6) = \\
\{a\}, \text{red}(7) = \{b\}, \text{red}(8) = \{d\}, \text{red}(9) = \{e\}, \text{red}(10) = \{b\}, \text{red}(12) = \{a\}, \)

(d) ![Diagram](attachment:image.png)

Solution 6.2
We define actions \(a_0, a_1, a_2, b_0, b_1, b_2,\) and \(c_0\) for statements in \(m, n,\) and \(p,\) respectively. Each state in the Kripke structure is a tuple of program locations and a valuation of \(g.\) Notice that it is not necessary to explicitly models valuations of \(x\) and \(y\) as they are implicitly defined by program locations of \(m\) and \(n.\)

For each property, we construct a labeled Kripke structure \(K = (S, A, \rightarrow, r, AP, \nu),\) where and \(S, A, \rightarrow,\) and \(r\) are as follows:
The independence relation $I = (A \times A \setminus Id) \{ (b_2, c_0), (c_0, b_2) \}$.

Next, we consider each property individually.

a) The corresponding LTL formula is $F(g == 1)$, where $AP_a = \{ g == 1 \}$. So, $\nu_a(s) = \{ g == 1 \}$ iff the valuation of $g$ in the state $s$ is 1, and as a result, $U = A \setminus \{ b_2, c_0 \}$. A possible reduced Kripke structure is as follows:

```
(\begin{array}{llll}
(m_0, n_0, p_0, 0) & a_0 & (m_1, n_0, p_0, 0) & a_1 \\
(m_2, n_0, p_0, 0) & a_2 & (m_3, n_0, p_0, 2) & b_0 \\
(m_3, n_1, p_0, 2) & b_1 & (m_3, n_2, p_0, 2) & c_0 \\
(m_3, n_2, p_1, 1) & (m_3, n_3, p_0, 0) & (m_3, n_3, p_1, -1)
\end{array})
```
b) The corresponding LTL formula is $m_1 \mathbf{R} \neg n_3$, where $AP_b = \{m_1, n_3\}$. $\nu_b(s) = \{m_1\}$ (resp. $\{n_3\}$) iff the $s$ contains $m_1$ (resp. $\{n_3\}$). As a result, $U = A \setminus \{a_0, a_1, b_2\}$. A possible reduced Kripke structure is as follows: