Model Checking – Exercise sheet 10

Exercise 10.1
Consider the following Kripke structures $K_1$, $K_2$, and $K_3$, over $AP = \{ p \}$:

(a) Does $K_2$ simulate $K_1$? If yes, give a simulation relation. Otherwise, explain why.
(b) Does $K_2$ simulate $K_3$? If yes, give a simulation relation. Otherwise, explain why.
(c) Does $K_3$ simulate $K_2$? If yes, give a simulation relation. Otherwise, explain why.
(d) Does $K_3$ simulate $K_1$? If yes, give a simulation relation. Otherwise, explain why.

Exercise 10.2
Let $K_1$, $K_2$, and $K_3$ be Kripke structures. Show that if $K_1$ and $K_2$ are bisimilar, and $K_2$ and $K_3$ are bisimilar, then $K_1$ and $K_3$ are also bisimilar.
Exercise 10.3
(Taken from ‘Principles of Model Checking’)

Let $TS = (S, Act, \rightarrow, I, AP, L)$ be a transition system. A bisimulation for $TS$ is a binary relation $R$ on $S$ such that for all $(s_1, s_2) \in R$:

- $L(s_1) = L(s_2)$.
- If $s'_1 \in Post(s_1)$, then there exists an $s'_2 \in Post(s_2)$ with $(s'_1, s'_2) \in R$.
- If $s'_2 \in Post(s_2)$, then there exists an $s'_1 \in Post(s_1)$ with $(s'_1, s'_2) \in R$.

States $s_1$ and $s_2$ are bisimulation-equivalent (or bisimilar), denoted $s_1 \sim_{TS} s_2$, if there exists a bisimulation $R$ for $TS$ with $(s_1, s_2) \in R$. The relations $\sim_n \subseteq S \times S$ are inductively defined by:

(a) $s_1 \sim_0 s_2$ iff $L(s_1) = L(s_2)$.

(b) $s_1 \sim_{n+1} s_2$ iff

- $L(s_1) = L(s_2)$,
- for all $s'_1 \in Post(s_1)$ there exists $s'_2 \in Post(s_2)$ with $s'_1 \sim_n s'_2$,
- for all $s'_2 \in Post(s_2)$ there exists $s'_1 \in Post(s_1)$ with $s'_1 \sim_n s'_2$.

Show that for finite $TS$ it holds that $\sim_{TS} = \bigcap_{n \geq 0} \sim_n$, i.e., $s_1 \sim_{TS} s_2$ if and only if $s_1 \sim_n s_2$ for all $n \geq 0$. 