Exercise 1 (Hypergraphs)

A hypergraph extends the concept of a graph in the sense that edges are allowed to connect an arbitrary number of vertices (instead of exactly two). Hence, a hypergraph is defined as a tuple \((V, H)\), where \(V\) is a set of vertices and \(H\) is a set of hyperedges, where \(H \subset \mathcal{P}(H) \setminus \{\emptyset\}\), with \(\mathcal{P}(H)\) the power set (i.e., the set of all possible subsets) of \(H\).

1. Give a suitable definition of the concept of a path in a hypergraph.

2. Now, consider the hypergraph \(S = (V_S, H_S)\) of “all” scientific articles, where \(V_S\) is the set of all authors, and each hyperedge \(h \in H_S\) contains all authors of a specific scientific article. The Erdős number \(Er(a)\) of an author \(a \in V_S\) is defined as the length of the shortest path in \(S\) that connects the specific vertex \(e \in V\) (\(e\) corresponds to the author Paul Erdős) to \(a\). Write down an algorithm to determine \(Er(a)\).  
   **Hint:** Think about how graph algorithms presented in the lecture might help you here.

3. Try to formulate the above problem as a graph problem, i.e. given \((V, H)\) construct some graph \(G\) such that the solution of a particular problem on \(G\) gives you the Erdős number.