Quantitative Verification – Exercise sheet 8

Exercise 8.1
Prove that a finite DTMC is irreducible iff its induced graph is strongly connected.

For the remaining questions, we define the following notation $F_{\sim p} \phi := P_{\sim p}[F \phi]$, and $G_{\sim p} \phi := P_{\sim p}[G \phi]$ for $\sim \in \{=, <, \leq, >, \geq\}, p \in [0, 1]$, and $\phi \in \mathsf{PCTL}$. We define analogous abbreviations for the step-bounded versions of $F$ and $G$.

Exercise 8.2
Translate the following formulae to English

1. $\text{send} \Rightarrow F_{\leq 0.95} \text{deliver}$
2. $P_{\leq 0.95} F_{\geq 0.9} \text{error}$
3. $P_{\geq 0.8} \text{empty} U (\text{send} \land G_{\leq 0.5} \neg \text{receive})$

Exercise 8.3
Translate the following specifications into $\mathsf{PCTL}/\mathsf{PLTL}$ formulae

1. The system with two processes satisfy mutual exclusion almost surely ($\text{crit}_i$ holds if process $i$ is in the critical section).
2. The probability that every request will eventually be granted with a probability greater than 0.95 is 0.99.
3. The probability that component $B$ fails ($B_{\text{fail}}$) before component $A$ ($A_{\text{fail}}$) is less than 0.4.

Exercise 8.4
For each of the following properties, draw a labelled Markov Chain which satisfies it or argue why the property is unsatisfiable.

- $G_{\leq 0.5} (a \land \neg b)$
- $G_{= 1} (-a \land F_{= 1} a)$
- $-a \land P_{\geq 1} [b U a]$
- $F_{= 1} (a \Rightarrow (G_{= 1} ((b \Rightarrow c) U -a \land (F_{\geq 0.5} c \lor \neg b))))$